

Fostering pre-service teachers' noticing of mathematical thinking through textual AI-human interactions: Opportunities and challenges

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This paper explores pre-service mathematics teachers' (PTs) responses to students' thinking during AI-based teacher-student simulated textual interactions. In this pilot study, critical events are used to facilitate teacher noticing (i.e., identifying, interpreting, and responding to students' thinking) where an AI system (i.e., Copilot) acts as a student. Data consists of eight PTs' transcripts of prompt history around a critical event, focusing on dividing an equation by x . We analyzed the transcripts using the concepts of "disregarding" and "shaping," which revealed mathematical ideas pre-service teachers responded to and disregarded when discussing with an AI system. Most PTs responded to the ideas of dividing the equation by x and factorization, while disregarding the idea that dividing by x was transferred from a previous topic. We show the variation in participants' interactions and discuss implications for fostering teachers' ability to notice critical events.

Keywords: AI-based teacher training, pre-service teachers, mathematics teacher education, teacher noticing, critical events.

Introduction

Effective mathematics teaching requires fostering teachers' responsiveness skills, such as teacher professional noticing, which enables teachers to attend to and effectively respond to students' statements, thereby fostering deeper understanding (e.g., van Es & Sherin, 2002). These noticing skills, comprised of identifying, interpreting, and responding to students' thinking, can be learned, and many teacher education programs position them as central pillars in the programs (e.g., Amador et al., 2021). Research shows that when learning to notice students' thinking, PTs manage to learn to identify students' thinking and develop their interpretations of that thinking. Yet they experience challenges in responding to the thinking they just interpreted. Often, the PTs' suggestions for responding to students' thinking are oversimplified and disconnected from the mathematics that is anchored in the students' thinking (e.g., Rotem & Ayalon, 2024). Thus, more meaningful practice opportunities are needed to support PTs' nuanced ways of responding to student thinking.

Current research suggests that GenAI can provide new prospects for simulating such practice opportunities (Dieker et al., 2024). In the rapidly growing field of using GenAI to support PTs' learning to notice students' thinking, most research examines teacher noticing using pre-post tests after the teachers interacted with the Chatbot (e.g., Galiç et al., 2025). Here our goal is to qualitatively explore how AI-based teacher-student interactions can support this endeavor, exploring the nuanced ways GenAI can support PTs to respond to student thinking. To address this goal, we developed an AI-based learning activity that simulates student-teacher dialogues around student thinking through textual interactions. The PT assumes the role of the teacher, while an AI system simulates the student. The AI acted based on common student difficulties and misconceptions across various subjects, as evident in academic literature (e.g., Welder, 2012). This pilot study is a part of a larger research

project aimed at investigating the opportunities and challenges of leveraging AI systems to support PT in noticing student thinking.

Literature background

Teacher noticing has expanded notably over the past two decades, developing well beyond its original conceptualization. One preliminary conceptualization, introduced by van Es and Sherin (2002), defines teacher noticing as identifying students' mathematical thinking during classroom situations and making knowledge-based interpretations of it. Later conceptualizations introduced responding to student thinking in ways that promote their mathematical understanding as an inherent additional skill of teacher noticing (Jacobs et al., 2010). Rotem and Ayalon (2024) use the construct of *critical events* to support PTs' learning to notice students' mathematical thinking as it unfolds during the lesson. Critical events are moments in which students' thinking becomes apparent and can serve to deepen students' mathematical understanding. They demonstrated that while PTs become better at identifying and interpreting students' thinking, PTs often struggle to respond to student statements in ways that build on previously interpreted students' thinking.

Most research on teacher noticing has been conducted using videotaped excerpts and pen-and-paper tests (e.g., Amador et al., 2021). However, teacher noticing skills are practice-based enacted skills during a lesson. As such, to fully comprehend how such skills are learned and developed, they need to be taught and assessed while the teacher practices them during teacher-student interactions. Examining teacher noticing in practice – as teachers practice their noticing skills – holds some conceptual and methodological challenges. For example, distinguishing whether a teacher's question following a student's comment is meant to scaffold the student's thinking (thus building on their comment) or gain more information about the student's intentions (thus supporting the teacher's interpretation), can be subject to the researcher's interpretations (e.g., van Es & Sherin, 2021).

Addressing this challenge, van Es and Sherin (2021) expanded their conceptualization of teacher noticing and included additional constructs: *disregarding* and *shaping*. Disregarding is the other side of the coin of identifying. While “identifying key features of instruction” (van Es & Sherin, 2021, p.19), selected features of classroom interactions are being *disregarded*, to allow the teacher to focus on aspects they find more relevant in terms of students' thinking. Shaping is defined as a set of “construct interactions and contexts that provide access to additional information” (ibid., p.19). Through shaping, teachers are taking an inquiry stance and engaging with students' thinking to gain a better understanding of their thinking. Hence, in the teacher-student interaction, some of the teacher's statements can be part of their response to support students' understanding. In contrast, other statements can, in fact, be part of the teacher's shaping and questioning of the critical event.

We build on the recent theoretical development in teacher noticing described above to address our goal and explore the nuanced ways GenAI can support PTs to respond to student thinking. To do so, we designed, implemented, and evaluated a workshop where eight PTs interacted with a GenAI system around a critical event showcasing a moment in which the teacher can build their instruction on the students' understanding. Consequently, we ask: *In what ways can engaging in AI-based textual teacher-student interactions support PTs' responding to student thinking?*

Methods

Research setting and context

This research is situated in the context of a teaching methods course at Utrecht University. All course participants hold a BSc in mathematics or a comparable degree in terms of mathematical content knowledge. The methods course, co-taught by the first author, is part of a master's program designed to prepare PTs to teach mathematics at the upper-secondary level. This course is accompanied by a one-day-per-week internship. Topics covered in the course include learning and teaching functions, derivatives, integrals, and several overarching themes, such as problem-solving and argumentation. One session was devoted to teacher noticing of critical events.

The study was conducted during a workshop held after the course concluded, to which participants voluntarily enrolled. The voluntary character of the workshop enabled a low-stakes participation and participants having completed the course while following good research practices (McGill et al., 2023). All eight course participants participated in the workshop. The workshop focused on several critical events from secondary school mathematics classrooms, providing PTs with multiple opportunities to engage with student-like thinking that emerged within these events. In this study, we focus on the case of a critical event that is often encountered as a challenge in mathematics education and can provide a rich context for teacher noticing (Rotem & Ayalon, 2024), the critical event is:

“The critical event happened in a 9th-grade mathematics lesson. While solving the equation: $x^3 = 6x^2 - 5x$, the student attempted to solve the equation by writing down the next step as: $x^2 = 6x - 5$.”

The event was based on a critical event reported by a teacher who participated in the first author's previous project (Rotem & Ayalon, 2024; project webpage: <http://aklim.haifa.ac.il/english>). This critical event, along with the developed material containing GenAI prompts, was explicitly designed to capture a frequently occurring critical event, such as when students learn to solve polynomial equations of degree three and higher, among others. Mathematical ideas that may arise during the solving process include: recognizing that x can be zero (and is zero in this case), understanding that division by zero is undefined, and applying factorization to the trinomial expression. Conceptual ideas can also be raised, such as generalizing a previously learned solving strategy. The PTs copy-pasted the critical event and a pre-designed prompt into Microsoft Copilot, an AI assistant integrated into Microsoft 365 applications, provided by the university. Using the university's licensed Copilot application helped mitigate privacy and security concerns associated with unregulated GenAI tools.

Data collection and analysis

Data collection consists of the eight transcriptions of PTs' interactions with the Copilot application around the critical event mentioned above. All prompts were written in English, and the PTs could choose whether they wished to interact with the Copilot application in English or Dutch. Six participants interacted in English, and two in Dutch.

The unit of analysis is turn-talk. The average number of turn-talks in the AI-based teacher-student interactions is 16 turns, with a standard deviation of 3 turns. Firstly, for each turn, we identified the mathematical ideas anchored within that turn, for both the AI system (Copilot) and the PTs. Secondly,

we deductively coded the PTs' responding turns using 'disregarding' and 'shaping' (van Es & Sherin, 2021). If a mathematical idea was raised by the Copilot-student and then addressed by the PT, then that mathematical idea was considered as noticed by the PT. Mathematical ideas raised by the Copilot-student and ignored by the PT were coded as 'disregarded'. For all PTs' responses, we also examined whether these responses were questions that provided access to additional information about the students' thinking, thus 'shaping' or whether these were statements and questions that aimed to guide the Copilot-student toward the right solution strategy for the task. Table 1 depicts the coding scheme. After coding both the Copilt-student-like turns and the PT, we counted frequencies for mathematical ideas that were: a) noticed, and b) disregarded by the PTs. We also explored how much and in what ways the PTs shaped the interaction.

Table 1: Codebook - responding to the Copilot-student

Code	Definition
Noticing mathematical ideas taken by the PT	Mathematical ideas taken by the PT indicate that the PT <i>identifies</i> these mathematical ideas, <i>interprets</i> them, and makes a conscious decision to <i>respond</i> to them.
Disregarding mathematical ideas	Mathematical ideas that are raised by the Copilot-student and are not taken by the PT
Shaping	Questions providing access to additional information about the student's thinking
Directions to carry on the solution	Statements or other guiding questions (such as rhetorical questions)

Results

Figure 1 depicts the mathematical ideas that emerged throughout the eight AI-based teacher-student interactions' transcripts, while indicating noticed/disregarded mathematical ideas.

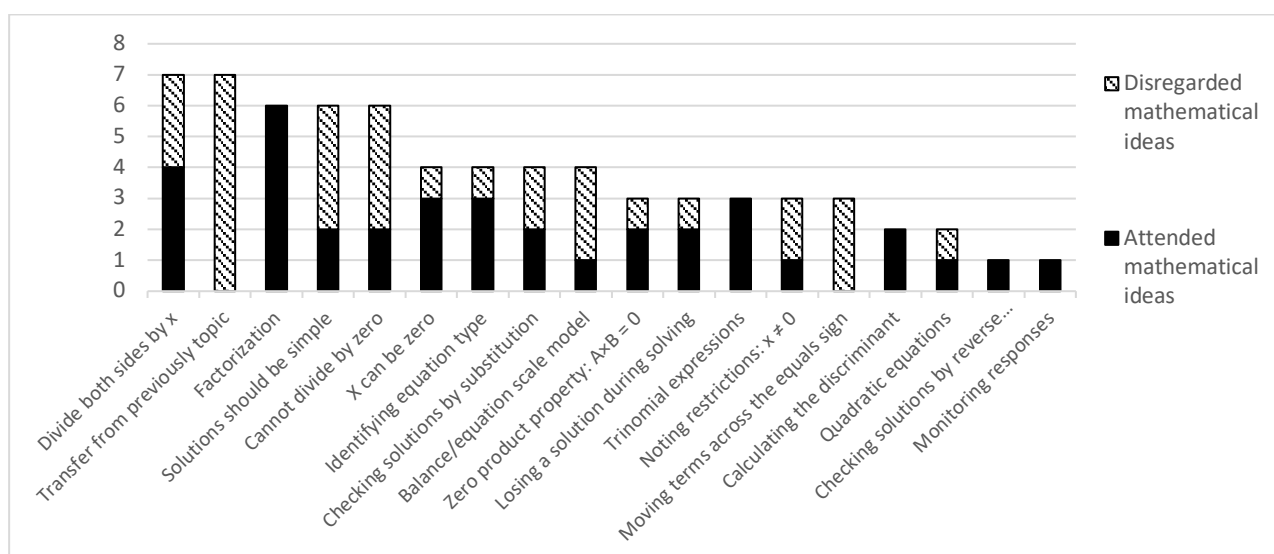


Figure 1: Mathematical ideas attended, and disregarded by the PTs during their AI-based interactions

Our data analysis as illustrated in Figure 1, shows that the most prominent mathematical ideas that were raised during the interactions were: a) dividing both side of the equation by x (7 scripts out of 8); b) transferring strategies from previously learned topic (i.e., using ‘old’ strategies in ‘new’ tasks) (7 scripts out of 8); c) factorization (6 out of 8); d) cannot divide by zero (6 out of 8), and e) solving should be easy (6 out of 8). The ideas that were predominantly noticed by the PTs in the interaction are: a) dividing both sides by x ; b) factorization; c) x can be zero, and d) identifying equation type. These ideas align with the expected solution strategy. Ideas that appeared in more than half of the transcripts and were mostly disregarded by the PTs were: a) a solution strategy can be transferred from a previously learned topic; b) solutions should be simple; c) one cannot divide by zero; and d) the balance/equation scale model.

Additionally, most of the PTs’ responses to the Copilot-student were to give directions for carrying out the solution, while fewer were to shape the interaction, i.e., using questions to provide access to additional information about the student's thinking. These findings are depicted in Table 2.

Table 2: PT’s type of responses to the Copilot-student in the AI-based interaction

PT #	Total number of turns	# PT's turns	Giving directions to carry out a solution (# turns)	Shaping moves (# turns)
PT1	11	5	5	0
PT2	15	8	6	2
PT3	20	10	9	1
PT4	12	6	5	1
PT5	15	7	7	0
PT6	20	10	9	1
PT7	17	8	6	2
PT8	15	7	7	0

From Table 2, together with Figure 1, it emerges that the PTs mainly used the Copilot-student ideas to give direction to move forward with the task solution. Still, the AI-based teacher-student interactions provide PTs with the opportunity to practice their skills of responding to student thinking. Furthermore, the variety of mathematical ideas emerging from these transcripts demonstrates the variations in the transcripts, providing different ways of learning to notice, as we will exemplify next.

Variations among PTs’ transcripts in the AI-based teacher-student interactions

We illustrate the variation between PTs’ transcripts in AI-based teacher-student interactions with two examples: excerpts from PT2’s and PT4’s AI transcriptions. The emojis appearing in the text were part of the AI-based interaction, marked as Copilot. First is PT2’s example showing 6 out of 15 turns.

Student (me, raising hand): Um, excuse me, sir? I think I messed up somewhere... I was trying to solve this equation: $x^3 = 6x^2 - 5x$. And then I thought I could just divide both sides by x , so I wrote: $x^2 = 6x - 5$. But now I'm not sure if that's right? I thought you could [...] that, 'cause we did something like that in another problem.

PT2: Hi there, you didn't do it completely wrong, but you forgot something.

Copilot: Ohhh... wait, so I was kinda close? 🤔 Is it because I divided both sides by x , but I didn't think about what happens if $x = 0$? Like... I just canceled the x 's without checking if that's allowed. But I thought we could always divide both sides by the

- same thing? So... should I just divide everything by x again but like... write " $x \neq 0$ " next to it? Or is there a better way to do it? Also, what did I forget exactly?
- PT2: Do you remember how to solve $x^2 - 4x = 0$?
- Copilot: Ohhh yeah! I think I kinda remember that one! 😊 So for $x^2 - 4x = 0$, I think we did something like... take out the x ? Like factor it? So it becomes: $x(x - 4) = 0$ And then we said something like, "if two things multiply to zero, then one of them has to be zero," so we did: $x = 0$ or $x - 4 = 0$, which means $x = 4$... Wait... is that what I was supposed to do with the $x^3 = 6x^2 - 5x$ one too?? Like factor it instead of dividing? But I thought dividing was faster... 😊
- PT2: Exactly! you are on the right track. So why didn't you divide by x in the last example? As you did in the first example?

In the first turn, the Copilot-student turns to PT2 and raises two mathematical ideas: dividing both sides by x , and that a solving strategy can be transferred from previously learned lesson. The Copilot simulating the student also indicates insecurity regarding their solution strategy. PT2 first reassured the student and then hinted that something was forgotten. Then, the Copilot-student raised the idea that x can be zero, that we cannot divide by zero, the scale model of the equation, and that restrictions should be noted. PT2 referred to a similar equation, which is somewhat simpler, and *disregarded* all these mathematical ideas. By asking about a similar equation, PT2 *shaped* the critical event, as this question provided access to additional information about the student's thinking. This leads the Copilot-student to suggest factorization, a central concept needed for solving this equation.

Next is PT5's transcript with the Copilot-student. PT5 also used shaping, yet it led the conversation in a different direction. This excerpt shows turns 2-5 of the transcript (out of 15). Turn 2 is the first PT's reply to the Copilot-student request for help.

- PT5: Hi, yes indeed, if you divide by x and x turns out to be zero, you did something that wasn't allowed. But maybe you can do something similarly, called binomial factoring, do you remember what that is?
- Copilot: Ohhh right! Binomial factoring... I think I kinda remember that from last week? 😊 Is that the thing where you look for something that all the terms have in common and then you pull it out in front of parentheses? Like... a common factor? So for $x^3 = 6x^2 - 5x$, maybe I could factor out an x from everything? Like: $x(x^2) = x(6x - 5)$. Wait... is that right? I just put x in front of both sides because they all have x in them. But now I'm not sure what to do next... Also... is this even binomial factoring or did I just make up a new thing? 😊
- PT5: Yes, indeed, like a common factor! This is similar to binomial factoring, you're almost there. What do you do when you try to solve an equation like this?
- Copilot: Ohhh okay, so I was kinda close! 😊

This excerpt starts with PT5's first turn occurring right after the Copilot-student turn, which was similar to the first turn with PT2 (due to the prompt). Yet, PT5 raised the idea that x can be zero, and division by zero is then undefined (or, in her words, it is not allowed). In that respect, she built on the Copilot-student statements, while directing it to the desired solution strategy by raising the idea of factorization. The Copilot-student's factorization was done in an interesting way, which would potentially provide an opportunity for the PT to consider a possible follow-up response. Here, PT5 mentions a common factor and encourages the student to proceed with factorization. She does not give the solution, and her question seems to guide the solving process. Yet, similar to the excerpts above, many of the Copilot-student ideas were *disregarded* by the PT5, such as the idea that solving

strategies can be transferred from previously learned topics. Nonetheless, this interaction provides an opportunity for PT5 to delve into the student's thinking.

Discussion

Overall, our analysis showed that the developed activity enabled PTs to engage in responding to student-thinking-like of the GenAI. We learned that the PTs noticed some mathematical ideas introduced by the Copilot-student, mainly involving dividing both sides of the equation by x and factorizations. These ideas are central mathematical ideas in this event. Yet, while noticing some ideas, the PTs disregarded others. Notably, all PTs disregarded the Copilot-student's reasoning for why they divided the equation by x ; that is, the Copilot-student claimed that they had transferred something from previously learned lessons. Possible reasons why the PTs disregarded this idea could be, first, that they perceived the idea as a conceptual aspect to which the PTs did not think they needed to respond or did not know how to respond. Another explanation is that once the Copilot-student presented a correct idea, PTs felt less compelled to return to other ideas, even if these held mathematical significance. Building on these findings, the variation that exists among PTs' transcripts in terms of the opportunities to respond to, disregard, and shape different mathematical ideas suggests that AI-based teacher-student textual interactions can provide valuable opportunities for PTs to practice responding to students' thinking.

Indeed, accessible AI systems like Copilot hold great potential as a complement to traditional learning environments, e.g., by tackling logistical barriers to teacher training and better preparing PTs for their internships, while providing teacher trainers with the ability to effortlessly check PTs' interactions (e.g., Galiç et al., 2025). Nonetheless, such activities require thoughtful design and facilitation to ensure they foster, rather than constrain, the development of teachers' noticing. Although the critical events in the workshop reflected common challenges documented in the literature and/or observed in PTs' classroom experiences, they still involved interaction with a computer. Even if participants perceived the experience as authentic, it did not constitute real teaching. Further, overreliance on AI in teacher training raises environmental and ethical concerns, as well as the risk of reinforcing a narrower focus on procedural correctness at the expense of conceptual engagement, thereby limiting the professional growth of PTs.

Like every study, this pilot study has several limitations that must be acknowledged—including the "clean" experimental setting, the AI's comprehensive knowledge base, and ongoing privacy concerns (Golda et al., 2024). Yet, the current widespread engagement of people with and support-seeking from GenAI across various domains (e.g., Brandtzaeg et al., 2022) suggests that teacher education, like other fields, could potentially benefit from GenAI. We argue that our work is a first step, as it indicates that there is substantial room for teachers to use AI systems to simulate their upcoming lessons. While considering our results as a starting point for learning about responding to students' thinking using AI systems, more research in this direction is needed. From a methodological and theoretical perspective, the dual lens of disregarding and shaping could potentially be elaborated beyond this study and provide valuable insights into analyzing teacher noticing in the midst of instruction, a field that remains under-researched despite its great importance for classroom teaching.

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